A NUMERICAL MODEL FOR MULTI-LEAF STONE MASONRY

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ABSTRACT
This paper presents a non-linear finite element material model for the analysis of the mechanical behavior of multi-leaf masonry, which is quite frequently used in ancient stone masonry buildings. A damage model, previously developed by some of the authors for brittle materials (namely, concrete) was adapted to fit the experimental stress–strain behavior of stone masonry. To this end, a damage evolution law originally proposed for concrete was modified and a new material parameter was added. A distinguishing feature of this model is that damage is characterized by a second-order tensor, thus allowing oriented 'cracks' to be described; the orientation of the cracks is fixed once they are activated, whatever the subsequent stress history is. Then, the modified model was implemented into a subroutine, linked to a commercial finite element code suitable for nonlinear analyses (FEAP). In order to validate the obtained material model, results available from tests on three-leaf stone masonry prisms were employed. The damage evolution law was calibrated according to results obtained from the single leaves, individually tested. The numerical nonlinear structural response was obtained by assuming suitable displacements boundary conditions and employing a tangent stiffness matrix procedure. In nearly all the applications, the finite element analyses predicted the experimentally measured peak load with good accuracy. However, the post-peak behavior was not always satisfactorily described: this is likely to be attributed to some numerical instability, which will be overcome in a future version of the model.

1 INTRODUCTION

The knowledge of the failure load for a composite specimen using numerical procedures based on the characteristics of the components is important and useful. This is true because of two main reasons. First, it is usually much easier to test the individual components rather than the composite specimen. Second, it is possible to predict the failure load for many arrangement types using only theoretical procedures, avoiding expensive laboratory tests on large specimens.

The main purpose of the paper is to show that a suitably developed theoretical model can predict the failure load and, in most cases, also the failure displacements experimentally obtained for multi-leaf stone masonry specimens using a numerical procedure based on the stress-strain curves obtained for the single leaves.

The aim of the research is to define the mechanical behavior of multi-leaf masonry under vertical actions, in order to better characterize short and long term damage in this type of walls.

2 EXPERIMENTAL PROGRAM

The experimental program has been developed at Politecnico di Milano, Binda *et al*. [1]. Both single-leaf specimens, composed by only one material, and multi-leaf specimens, composed by two different materials, were tested. The multi-leaf specimens were built with two external leaves composed by stones with horizontal and vertical mortar joints, and an internal leaf, made by irregular pieces of the same stone layered with mortar. The geometry of the multi-leaf specimens is shown in figure 1.

Two types of stones were used: Pietra Serena (a medium grained sandstone) and Pietra di Noto (a limestone used in Sicily). All the tests were carried out under displacement control in order to follow the specimen behavior beyond the ultimate load. For each type of stone, three different types of tests were performed, differing in terms of loading conditions and geometry of the interface between internal and external leaves (with and without offsets). The different specimens are shown in figure 2: (*i*) type 1 tests are compression tests on specimens with continuous vertical interfaces between the inner and the outer leaves; (*ii*) type 2 tests are compression tests on specimens with offseted layers; type 3 tests are shear tests on specimens with offseted layers. The specimens with continuous vertical interfaces subjected to shear tests failed at mechanically meaningless load values, so that the results of these tests are not presented in this work.

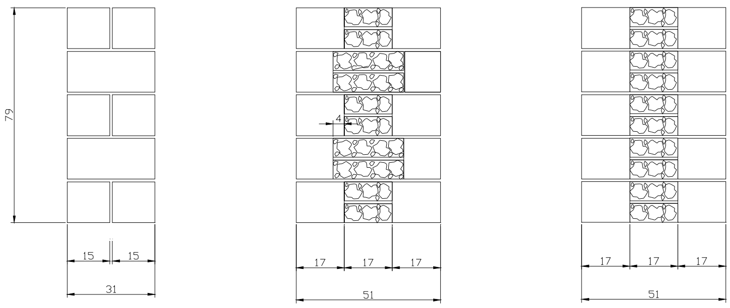
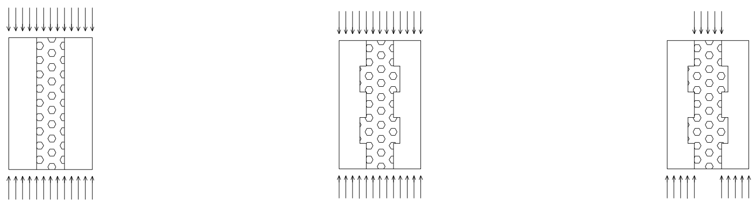


Figure 1 – Three-leaf specimen dimensions



(a) Type 1 (b) Type 2 (c) Type 3

Figure 2 – Specimen types

3 METODOLOGY FOR COMPARING RESULTS

All the numerical results have been achieved using 8-noded three-dimensional finite elements. A non-linear material model, allowing for the description of mechanical damage, was considered (see next item for further details). The non-linear structural response was obtained prescribing suitable displacements boundary conditions and employing a secant rigidity matrix procedure in order to represent the softening behavior. First, the mechanical parameters for each stone masonry type were identified using the proposed damage law, eqn (5). Then, the capability of the proposed FE model to describe the mechanical response of the individual, homogeneous leaves in simple compression was checked using a 960-elements mesh. Finally, the tests on multi-leaf specimens described in the preceding item were simulated using 2880-elements meshes in order to check the validity of the proposed model.

4 ORIGINAL DAMAGE MODEL

The damage model employed to simulate and analyze the experimental tests performed on single and multi–leaf walls was previously developed by some of the authors and successfully applied to structural analyses of ancient masonry towers. Details about this model can be found elsewhere

(Papa & Taliercio [2, 3]). The model was originally conceived to describe the time evolution of damage in brittle materials, such as masonry, under both increasing and sustained stresses. Accordingly, during any time interval $\Delta t_{(i)} = t_{(i)} - t_{(i-1)}$ the strain increment produced by any 3-D stress increment applied to a material element can be expressed, in matrix form, as

$$\{\Delta\varepsilon_{(i)}\} = [C_{(i)}]\ \{\Delta\sigma_{(i)}\} + \{\Delta\varepsilon_{(i)}^{in}\}, \tag{1}$$

where the last term is the inelastic strain increment during $\Delta t_{(i)}$ due to the stress acting on the material element at $t_{(i-1)}$ and $[C_{(i)}]$ is the current flexibility matrix. This matrix can be expressed as the sum of three matrices:

$$[C_{(i)}] = [C_{(i)}^{el}] + [C_{(i)}^{M}] + [C_{(i)}^{K}] \tag{2}$$

The first matrix describes the instantaneous (elastic) response of the (damaged) material, the second one accounts for creep–induced damage, and the third one for the viscoelastic response of the material [2]. As the present research deals with monotonic tests on masonry samples, attention is focused only on the first matrix.

The virgin (undamaged) material is supposed to behave isotropically: E^M, ν denote its elastic modulus and Poisson's ratio, respectively. Damage is described by a symmetric, second–order tensor $\boldsymbol{D}$, whose eigenvalues and principal directions are indicated, respectively, by D_α and x_α ($\alpha = I$, II, III). Accordingly, in the most general case, the damaged material element behaves orthotropically: this is consistent with a proposal by Kachanov [4]. x_I, x_{II} and x_{III} define the local symmetry planes of the damaged material: once any principal direction of damage is activated, it is supposed to remain constant throughout the subsequent stress history, thus leading to a "non-rotating, smeared crack model".

In the reference frame of the principal directions of damage, the elastic flexibility matrix of the damaged material reads:

$$[\hat{C}_{(i)}^{el}] = \frac{1}{E^M}\begin{bmatrix} \psi_{I,I}^{-1} & -\nu\psi_{I,II}^{-1} & -\nu\psi_{I,III}^{-1} & 0 & 0 & 0 \\ & \psi_{II,II}^{-1} & -\nu\psi_{II,III}^{-1} & 0 & 0 & 0 \\ & & \psi_{III,III}^{-1} & 0 & 0 & 0 \\ & & & 2(1+\nu)\psi_{I,II}^{-1} & 0 & 0 \\ & symm. & & & 2(1+\nu)\psi_{II,III}^{-1} & 0 \\ & & & & & 2(1+\nu)\psi_{III,I}^{-1} \end{bmatrix}, \tag{3}$$

where $\psi_{j,k} = [(1-D_j)(1-D_k)]^{½}$ ($j,k = I$, II, III). This matrix can be rotated to get $[C_{(i)}^{el}]$ in any Cartesian reference frame using classical transformation laws.

In this model the damage–driving variable is supposed to be a 'damage force' tensor, which, by generalizing the definition given in the 1–D case by Lemaitre & Chaboche [5], is defined as $\boldsymbol{Y} = ½\ \boldsymbol{\varepsilon}^{el}\ \boldsymbol{\varepsilon}^{el}$. When the greatest eigenvalue of $\boldsymbol{Y}$ exceeds a threshold value at any point of the solid, the first damage direction (x_I) is activated. A second damage direction may be later activated in the plane orthogonal to x_I if the greatest direct component of the damage force tensor $Y_{\alpha\alpha} = \boldsymbol{n}_\alpha\cdot(\boldsymbol{Y}\boldsymbol{n}_\alpha)$, with $\boldsymbol{n}_\alpha \perp x_I$, exceeds the threshold values. Different threshold values (Y_{0T}, Y_{0C}) can be used according to the sign of the direct strain along any direction to match the different behavior of masonry in tension or compression. The damage law originally proposed to describe the experimentally observed behavior of the brittle materials such as concrete reads

$$D_\alpha = 1 - \frac{1}{1 + A_H \left\langle Y_{\alpha\alpha} - Y_{0H} \right\rangle^{B_H}}, \; \alpha = I, II, III \tag{4}$$

where A_H, B_H are material parameters, different for damage induced by tensile (H=T) or compressive (H=C) strains.

5 NEW DAMAGE LAW

The original damage law, eqn (4), was designed for concrete or brick masonry, less stiff than stone. Thus, it did not allow the behavior of stone masonry to be correctly reproduced. Accordingly, a new parameter was included in the damage law, which is now the following:

$$D_\alpha = 1 - \frac{C_H}{1 + A_H \left\langle Y_{\alpha\alpha} - Y_{0H} \right\rangle^{B_H}}, \; \alpha = I, II, III \tag{5}$$

The influence of the new parameter C_H on the stress-strain response of the material can be seen in Fig. 3; for the sake of illustration, an external leaf built with Serena stone is considered.

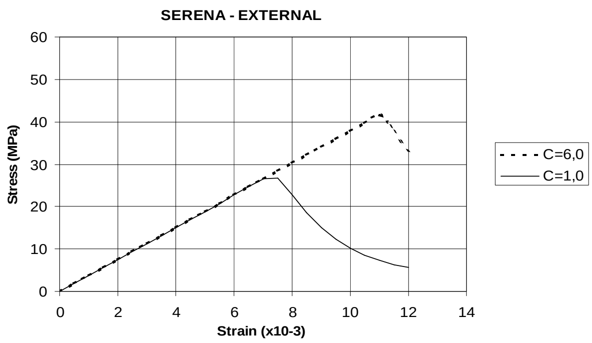


Figure 3 – Model sensitivity to parameter C

6 RESULTS

The results pertinent to the tests on single-leaf specimens are first shown. Figure 4 presents the strain-stress curves obtained for Noto stone, whilst in figure 5 results for Serena stone are depicted. For each stone, results for the internal leaf are at the left side, while the results for the external leafs are at the righ side. Experimental results (labeled Exp 1 and Exp2), analytical results (using eqn (5)), and numerical results (with 960 finite elements) are presented, in order to show the effectiveness of the model in reproducing the behavior of each material with the selected parameters. Note that the numerical results describe an excessively brittle post-peak behavior. This is likely to be a model's imperfection, mainly associated with strain localization, which will be overcome in future developments.

Concerning the multi-leaf specimens, the results are depicted in figures 6, 7 and 8 for types 1, 2 and 3, respectively. Note that the 'modified' numerical results were simply obtained by adjusting the raw ones to avoid the initial stage of the laboratory tests that has no significance. Anyway, the results are acceptable for most of the cases, in spite of the problems to correctly represent the post-peak softening behavior, as already pointed out with reference to tests on single-leaf specimens.

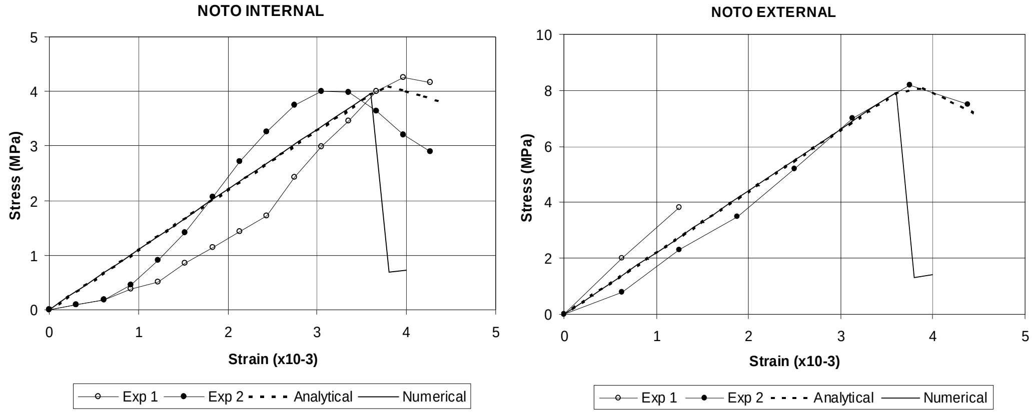


Figure 4 – Material characterization: Noto stone

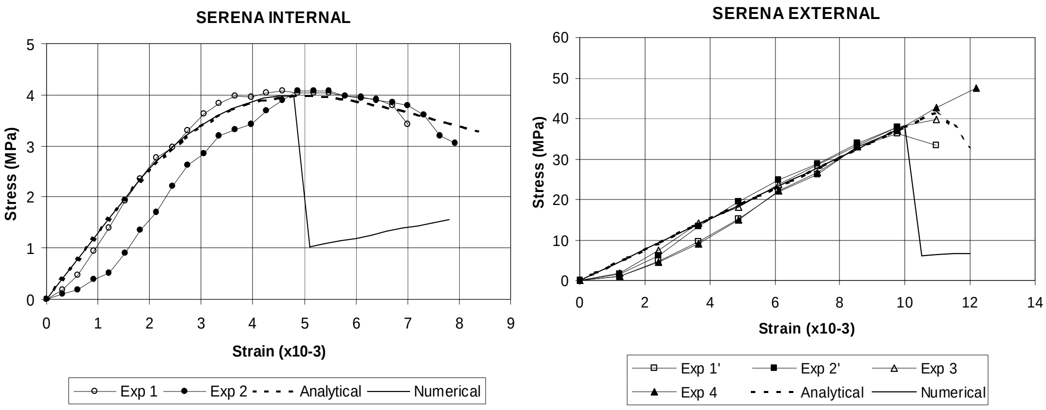


Figure 5 – Material characterization: Serena stone

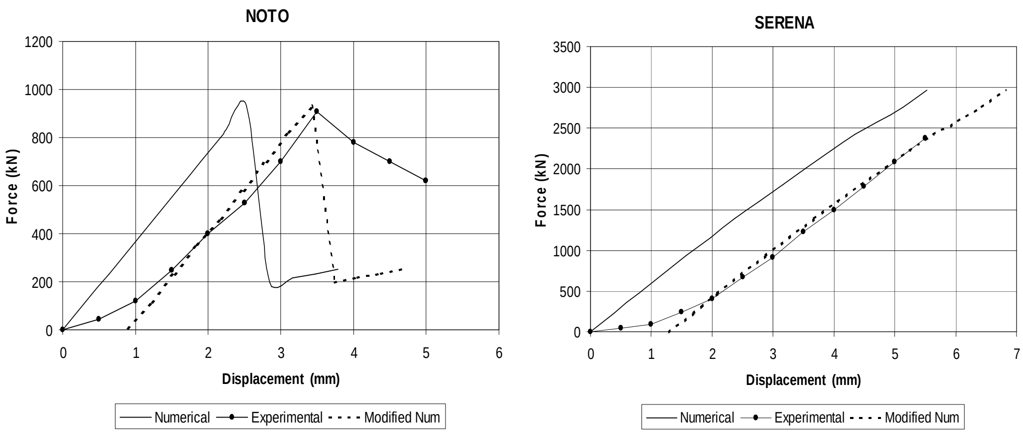


Figure 6 – Results for Type 1 tests

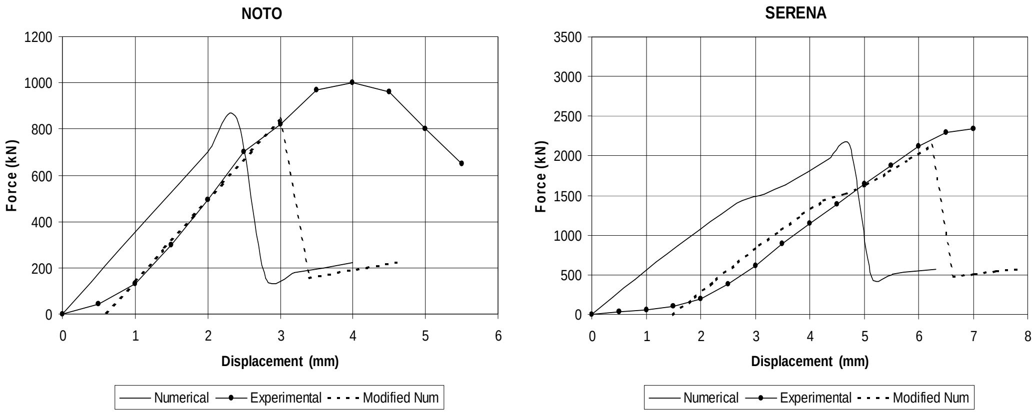


Figure 7 – Results for Type 2 tests

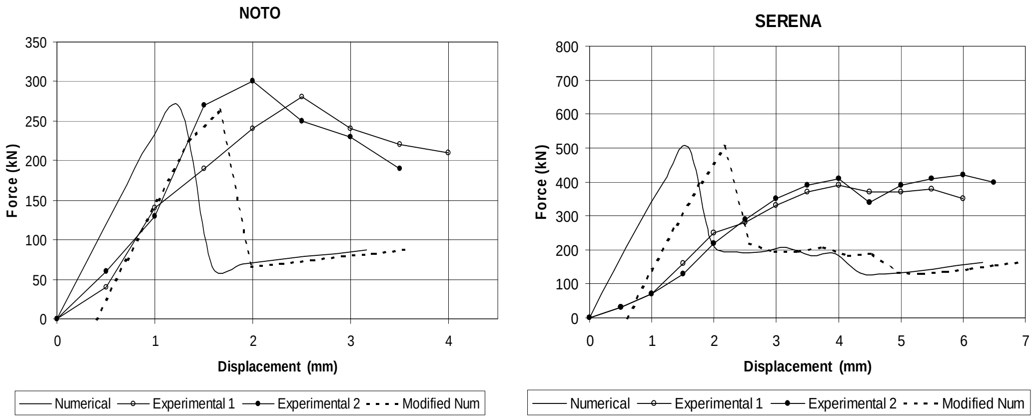


Figure 8 – Results for Type 3 tests

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